Theoretical and experimental validation of a non-linear vocal fold model

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Abstract. *The current paper explores the predictive capacity of a simplified two-mass model of phonation to reproduce non-linear characteristics for different initial parameter values by numerical model simulations. The influence of some one parameter variations on the predicted outcome is assessed. In particular, parameters related to the flow, mechanical and acoustical part of the model are adressed. Next, the predicted oscillation onset pressure is 'in-vitro' validated exploiting an experimental setup and deformable vocal folds replica for which the mechanical characteristics and the initial centre height can be imposed independently.*

1. Introduction

Simplified physical phonation models, like one or two mass models, aim to mimic the physiological fluid-structure interaction during vocal fold self-sustained oscillations with a limited number of parameters. The resulting reduced one or two degrees of freedom spring-mass-damper system allows to consider phonation as a dynamical system

$$\frac{dx}{dt} = F(x, \alpha), \tag{1}$$

depending on a set of parameters $\alpha \in \mathbb{R}^n$ and with $x, F(x) \in \mathbb{R}^n$. In general the equilibria satisfy a nonlinear system of equations $F(x, \alpha) = 0$ for which the solutions behaviour depends qualitatively on the parameters.

Before analysing the dynamic behaviour of reduced vocal folds models in terms of system singularities, the capacity of the two-mass model presented by Lous et al. (1998) to reproduce non-linear characteristics for different initial parameter values is explored by numerical model simulations and the predictive capacity of the model is 'in-vitro' validated towards experimental data obtained on a deformable vocal fold replica. Compared to a stability analysis of the physical model as in Ruty et al. (2007) numerical model simulations allow not only to validate oscillation onset characteristics, but in addition the dynamic behaviour is obtained and e.g. oscillation offset characteristics can be studied.

2. Physical modelling

Physical modelling of self-sustained oscillations of a vocal fold replica is performed by using a theoretical two-mass model and further exploiting the relationship between the input and output parameters in the physical model and the measured experimental variables. Briefly, the replica is modelled as a reduced spring-mass-damper system with two degrees of freedom driven by the pressure difference across the masses. The applied models for the airflow, vocal folds mechanics and acoustic interaction with a downstream waveguide are severe simplifications of the fluid-structure interaction during human voiced sound production. The distinct models and their interaction are inspired on the work presented in Lous et al. (1998) and are detailed in Ruty et al. (2007).

The airflow model relies on the assumption of a quasi-steady inviscid and incompressible flow within the glottis corrected for some major viscous effects, firstly in case of small vocal folds apertures and secondly to account for flow separation and the formation of a jet. The main parameters related to the flow model are the time evolution of the driving subglottal pressure $P_{sub}(t)$, the time evolution of the glottal cross sectional area $A(x, t)$ and the position of the separation point. The moving separation point is located in the diverging downstream part of the constriction at a position corresponding to the glottal area $A_s = c_s A_{min}$, with A_{min} the minimal constriction area and $c_s \geqslant 1$ an ad-hoc separation constant, following Liljencrants ad-hoc separation criterion. The requirement of the ad-hoc constant c_s is often argued in favour of more complex flow descriptions like boundary layer methods since the Liljencrants criterion does not adapt the value of c_s for a.o. the assessed volume flow velocity or the glottal opening height as e.g. shown in Van Hirtum et al. (2005) and Tanabe et al. (2006), respectively.

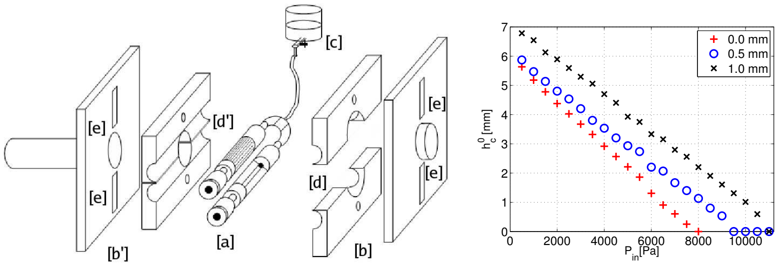
The vocal folds mechanics is modelled by the two mass model detailed in Lous et al. (1998); Ruty et al. (2007). The two mass model describes the movement of the masses rectangular to the flow direction assuming a rectangular glottal area with fixed width L_g the geometrical description can thus be given in terms of the centre height of the glottal area, i.e. $h_c = A/L_g$. Except geometrical parameters (e.g. the initial aperture h_c^0) and the subglottal pressure expressing the coupling with the flow model, the main parameters required in the mechanical model are the mass m, the spring stiffnesses K, the coupling stiffness between the two masses K_c, the damping R and the critical aperture h_{crit} applied in the discrete collision model. The relationship between the stiffness K and the coupling stiffness K_c is commonly expressed by introducing a parameter α, defined as $\alpha = \frac{K_c}{K}$. The influence of the mass m and the collision threshold h_{crit} on phonation features is shown in Ruty et al. (2007); Van Hirtum et al. (2006). Notice that in the current model the collision model is triggered on the glottal height, independent of the impact velocity.

The importance of the acoustical coupling between the vocal folds and a uniform downstream waveguide in the model depends on the first acoustical resonance frequency of the waveguide and therefore on its length L_d.

3. 'In-vitro' replica, setup and model parameters

The assessed deformable vocal folds replica and experimental setup is detailed in Van Hirtum et al. (2006). Small metal blocks, further referred to as separators, can be introduced between the upper and lower portion of the replica separating both latex tubes holders to a user controlled extent as indicated by [d] in Figure 1(a). This way the initial centre aperture at rest h_c^0 can be arbitrarily varied. The position of the upper and lower portion in previous replicas on the contrary, was fixed to a unique position corresponding to [d'] in Figure 1(a) (Ruty et al. (2007)). Consequently the initial aperture h_c^0 could only be varied by changing the internal pressure P_{in} in the latex tubes. P_{in} is controlled by lifting or lowering the water column labelled [c] in Figure 1(a). Concretely, separators of 0.0, 0.5 and 1.0 mm height are used, while P_{in} is incremented from 0 up to 11000 Pa by steps of 500 Pa. The resulting experimentally assessed (h_c^0,P_{in})-values in the parameter space are shown in Figure 1(b). The initial centre height h_c^0 varies significantly from complete closure for high P_{in} values to maximal values around 6 mm for low internal pressures. Notice that the exact value of h_c^0 depends indeed on the mounted separator. Therefore the advantage of the current replica is the independent variation of the quantities P_{in} and h_c^0 allowing to impose different initial conditions which is an important requirement in order to obtain a thorough experimental validation for the modelling of non-linear phenomena.

Next, the vocal folds replica is mounted in an experimental setup suitable to generate self-sustained oscillations and to study characteristic oscillation quantities. Apart from the discussed parameters h_c^0 and P_{in}, the following quantities are measured: the mechanical response of the replica in function of P_{in}, the deformation of the centre height of the replica $h_c(t)$, the supplied upstream pressure $P_{up}(t)$, the threshold upstream pressure corresponding to oscillation onset P_{on} and offset P_{off} and finally the downstream pressure $P_d(t)$ in case a downstream pipe with known length L_d and uniform diameter of 25 mm is attached to the replica. The latter allows to study the influence of the acoustical coupling between the replica and the waveguide on the self-sustained oscillations. In practice, three waveguide lengths L_d are considered, i.e. 50, 28.5 and 17 cm. The corresponding acoustical resonance frequency of the downstream waveguide yields 170, 300 and 500 Hz, respectively. Moreover, the length of the shortest waveguide approximates the length of an average male adult vocal tract , i.e. $L_d = 17$ cm.



(a) (b)

Figure 1. (a) schematic representation of the deformable replica, (b) centre apertures for separators 1.0, 0.5 and 0.0 mm as function of P_{in}.

In Ruty et al. (2007, 2006) a procedure is established in order to relate experimental control parameters to the required model parameters discussed in section 2. Except for the ad-hoc constant c_s the flow model parameters are directly related to measured quantities as e.g. the correspondence between $P_{sub}(t)$ and $P_{up}(t)$. The mechanical model parameters related to the spring and damper variables are obtained from the measured mechanical response of the replica and depend on the experimentally imposed P_{in}-value. The same way as for the constant parameter c_s in the flow model, the collision threshold h_{crit} is ad-hoc determined.

4. Results and discussion

In the following the influence of the distinct model parameters discussed in section 2 on the observed singularities is assessed. In section 4.1 the control parameters of the model are simulated in order to evaluate objectively the dynamic behaviour of the model with respect to the chosen parameter set. Next, in section 4.2 the dynamical model is validated on 'in-vitro' data generated with the experimental setup and vocal fold replica described in section 3.

4.1. Theoretical modelling and simulations

The model parameter sets discussed in section 2 for the flow $\{P_{sub}(t), A^0_{min}, c_s\}$, mechanical $\{h_{crit}, m, K, R, \alpha\}$ and acoustical $\{L_d\}$ model are in terms of experimental control and measurable quantities equivalent to $\{P_{up}(t), h^0_{min}, c_s\}$, $\{h_{crit}, m, P_{in}, \alpha\}$ and $\{L_d\}$, respectively. The model parameters K, R and α can be achieved by determining for each experimentally assessed P_{in} the mechanical resonance characteristics as mentioned in section 3. Table 1 summarises the default parameter values used as a benchmark in this section with P_{max} denoting the maximum upstream pressure $max(P_{up}(t))$. The given parameter values are chosen with relevance to the experimental data discussed in the next section 4.2. Besides the discussed parameters the time t in table 1 indicates the simulation duration. In the following one parameter will be varied from his reference value and the influence on the dynamic model behaviour is considered.

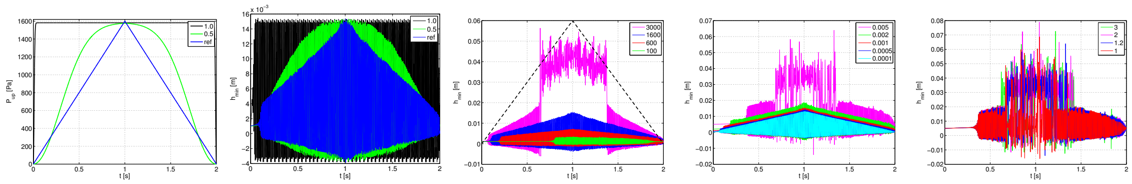
	flow			mechanical				acoustical
t [s]	P_{max} [Pa]	h^0_{min} [m]	c_s	h_{crit} [mm]	m [kg]	P_{in} [Pa]	α	L_d [cm]
2	1600	0.001	1.2	0.02	0.0001	9000	1.8	50

Table 1. Default modelling parameters for section 4.1.

4.1.1. Flow model parameters

At first, the driving upstream pressure $P_{up}(t)$ is considered. The upstream pressure is varied from zero to its maximum value following a symmetrical function increase and decrease. With respect to the function describing $P_{up}(t)$ in particular the variation of the maximum value P_{max} and the rate of variation are considered. The rate of variation can be modified by altering the simulation time, considering different $P_{up}(t)$ functions or a combination of both. Two functional descriptions are used. Firstly a triangular function and secondly an exponential function given as $P_{up}(t) = (1 - e^{-(at)^2})P_{max}$ with $0 \leqslant a \leqslant 1$ an arbitrary parameter determining the steepness to reach P_{max}. This is illustrated in Figure 2(a) for the parameter a set to $a = 0.5$ and $a = 1.0$, respectively. The influence of the functional variation shown in Figure 2(a) is illustrated in Figure 2(b). Next, the influence of varying P_{max} is given in Figure 2(c) for a triangular $P_{up}(t)$ indicated with a dotted line and P_{max} set to 3000, 1600, 600 and 100 Pa, respectively. Figures 2(d) and 2(e) show the influence of the initial aperture or the centre height h^0_c and of the separation criterion c_s ranging between 0.0005 and 5 mm and from 1 up to 3, respectively. The used parameter values largely cover the range of values commonly found in literature dealing with 'in-vitro' validation of physical phonation modelling.

The dynamic simulations of the glottal aperture illustrate the influence of the assessed flow parameters with respect to qualitative bifurcations characteristics. The influence is not limited to the occurrence of the Hopf bifurcation and the cyclic fold bifurcation with respect to oscillation on- and offset and therefore on the modelled qualitative hysteresis between oscillation on- and offset, but can be seen in the oscillation amplitude, duration and the occurrence of chaotic behaviour. The last is found to appear in case extreme parameter values as can be seen in Figure 2 for $P_{up} = 3000$ or $h_c^0 = 0.005$ m. The chaotic region is although interesting from a modelling point of view to be avoided in real life considering the resulting increased oscillation amplitudes. The depicted examples suggest that from a flow model point of view 'in-vivo' subjects develop control strategies to prevent these bifurcation regions by e.g. lowering the initial glottal aperture or the subglottal pressure, but here we need to appeal to other research fields in speech production familiar with the complex reality of 'in-vivo' subjects. Figure 2(e) can be commented in more detail, since although a slight influence on the bifurcation characteristics can be observed, the shown example is obained for $h_c^0 = 0.005$ m instead of the default parameter value of $h_c^0 = 0.001$ m for which no obvious influence appeared. Therefore the separation criterion seems to affect the dynamic behaviour only limited and this suggests that a moving separation point seems to be important, but the exact position seems less crucial.



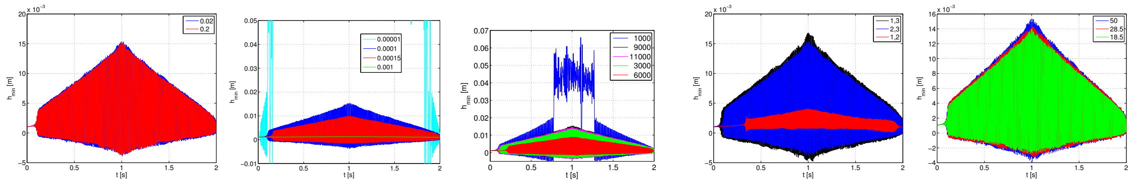
(a) $P_{up}(t)$ function (b) $P_{up}(t)$ (c) P_{max} (d) h_c^0 (e) c_s, h_c^0=5mm

Figure 2. Influence of flow model parameters.

4.1.2. Mechanical and acoustic model parameters

The influence of variations of the mechanical and acoustical model parameters on the dynamic behaviour of the system is assessed. Besides the parameters already discussed in subsection 4.1.1 Figure 3 illustrates the influence of the parameters h_{crit}, m, P_{in}, α and L_d, respectively.

A glance on Figure 3 illustrating the influence of the mechanical and acoustic parameters on the modelled glottal aperture reveals the impact of mechanical model parameters,



(a) h_{crit} (b) m (c) P_{in} (d) α (e) L_d

Figure 3. Influence of mechanical and acoustical model parameters.

corresponding to physical properties - i.e. the effective vibrating mass m and the mechanical resonance properties of the vocal folds set by P_{in} and α, on the modelled oscillation characteristics, i.e. oscillation duration, amplitude, on- and offset pressure thresholds, hysteresis effect and the occurrence of chaotic movement. It need to be noticed that the default value of the coupling parameter α given in Table 1 is defined on the first and second mechanical resonance frequency for $P_{in} = 9000$ Pa following the approach outlined in Ruty et al. (2006). Therefore the applied α value is slightly higher than the ad-hoc value classically used in physical two-mass vocal folds modelling where the coupling stiffness K_c is commonly set to half the spring stiffness K as $K_c = 0.5K$ corresponding to $\alpha = 1.4$, see a.o. Lous et al. (1998); Ruty et al. (2007); Van Hirtum et al. (2006). The observed behaviour with respect to the imposed P_{in} corresponds to the expected behaviour accounting for the mechanical resonance frequencies for P_{in} imposed to 1000, 9000, 11000, 3000 and 6000 Pa, respectively. The extracted features for the first mechanical resonance frequency, quality factor and α parameter yield $\{82, 6.8, 1.3\}$, $\{104, 9.5, 1.8\}$, $\{107, 10.7, 1.7\}$, $\{112, 8.6, 1.3\}$ and $\{141, 6.4, 1.0\}$, respectively. The same way as for the ad-hoc parameter c_s in the flow model, the ad-hoc threshold h_{crit} doesn't affect the mentioned features to a large extent considering moderate apertures h_c^0 away from closure as is the case for the assessed default parameter values summarised in Table 1. Finally the acoustic parameter L_d affects mainly the oscillation amplitude. The minor influence can be explained by looking at the mechanical resonance frequency of 100 Hz corresponding to $P_{in} = 9000$ Pa. The yielded frequency value is far below all assessed acoustical resonance frequencies, which explains that the influence of the acoustical resonance frequency is limited in the given example although the influence is shown in literature Lopez et al. (2006); Ruty et al. (2007).

4.2. 'In-vitro' experimental validation

'In-vitro' validation of the dynamic model is carried out as described in section 3. The experimental setup and the simulation results presented in section 4.1 enable and encourage to explore the influence of independent initial values, i.e. independent modifications of the initial centre height h_c^0 and mechanical properties by imposing different P_{in}. The assessed $\{h_c^0, P_{in}\}$ values are shown in Figure 1(b). In addition, the influence of the downstream waveguide length L_d is searched.

Figure 4 provides an overview of the estimated pressure thresholds for oscillation onset P_{on} and offset P_{off} for all experimentally assessed initial parameter configurations for h_c^0, P_{in} and L_d. The dashed line supposes equal on- and offset thresholds. Therefore Figure 4 clearly illustrates a predicted hysteresis with $P_{off} < P_{on}$. The same way as for the

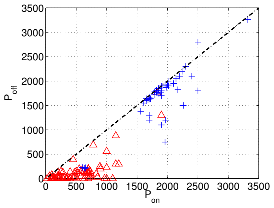
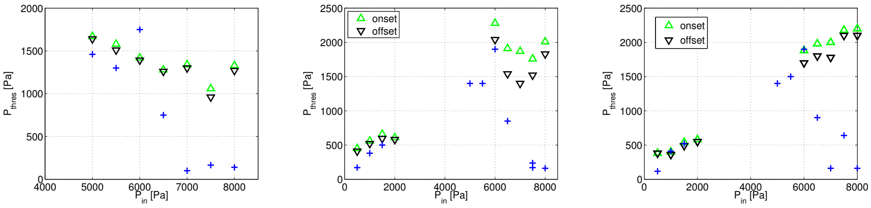
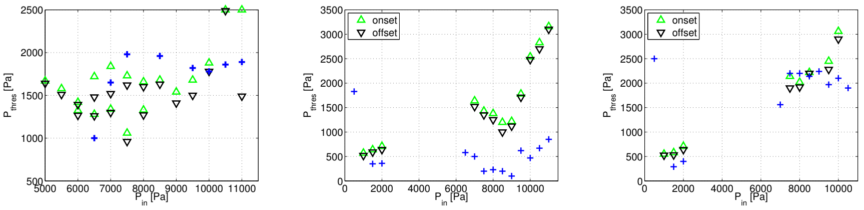


Figure 4. Overview of the predicted oscillation on- and offset upstream pressure thresholds for all assessed experimental configurations.

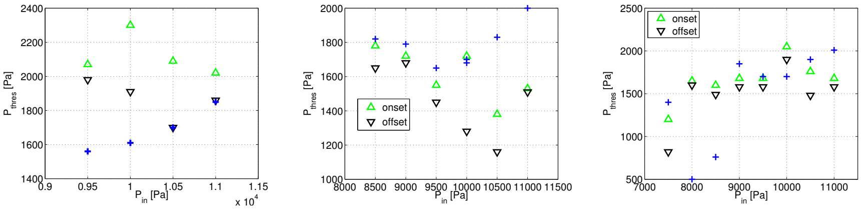
simulated data presented in section 4.1 two bifurcation regions can be distinguished. A first region involving threshold pressures up to 1000 Pa and a second region confined to much higher pressures from 1500 up to 2500 Pa. The two regions are also experimentally retrieved. Due to the interest for phonation in the following the validation of the predicted onset of oscillations P_{on} is inquired. Figure 5 illustrates the experimental and predicted results for variations of the separator, P_{in} and L_d, i.e. h_c^0, mechanical resonance properties and the acoustical resonance frequency in the modelling. For completeness also the experimentally observed P_{off} is given. In general the predicted P_{on} approximate qualitatively



(a) 0.0 mm, L_d = 50 cm (b) 0.0 mm, L_d = 28.5 cm (c) 0.0 mm, L_d = 17 cm



(d) 0.5 mm, L_d = 50 cm (e) 0.5 mm, L_d = 28.5 cm (f) 0.5 mm, L_d = 17 cm



(g) 1.0 mm, L_d = 50 cm (h) 1.0 mm, L_d = 28.5 cm (i) 1.0 mm, L_d = 17 cm

Figure 5. Experimental (△) and predicted (+) minimum pressure P_{on} in function of P_{in}, h_c^0 and L_d.

the experimentally observed values and this for all assessed P_{in} values. In order to obtain more accurate predictions for elevated P_{in} values, associated with closure of the replica, the discrete collision model should be revised as pointed out in Van Hirtum et al. (2006). Depending on the acoustical waveguide L_d and h_c^0 two oscillation regions are observed for the assessed P_{in}. This can e.g. be seen in Figure 5(b) where P_{on} values appear around 500 and 2000 Pa, respectively. Despite the satisfying qualitative outcome of the numerical model some caution in the interpretation is appropriate since the simulations are achieved based on either the first or the second mechanical resonance frequency in the modelling. Therefore it seems that depending on the imposed initial conditions the system prefers a distinct mechanical resonance frequency. It should be interesting to come up with a simplified physical model where no a posteriori decision on the mechanical resonance need to be made, but the choice is inherent to the model.

5. Conclusion

The performance of the two mass model presented in Ruty et al. (2007, 2006) for physical phonation modelling is evaluated. The influence of individual parameter variations in the flow, mechanical or acoustical part of the model reveals the influence on the model predictions. Dependent on initial conditions different bifurcations appear. This finding is in agreement with existing literature and confirms the non-linear nature of the phenomenon under study. Next, the independent modification of parameters is 'in-vitro' validated on a deformable replica. In particular the initial replica aperture and mechanical properties are varied in order to study the influence on the minimum pressure required to introduce self-sustained oscillations of the replica. The two mass model provides a qualitative explanation of the observed experimental data. However since the yielded predictions rely on the choice of the mechanical resonance as a model input it is interesting to develop a simplified physical model avoiding this a posteriori choice. Furthermore, a dynamical analysis of the validated model need to be performed and tested on experimental data.

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